EXCITED HEAVY MESONS FROM QCD SUM RULES

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Orbitally excited $L = 1$ charm mesons are studied by QCD sum rules in the framework of heavy quark effective theory. The meson masses and the strong decay widths are obtained. This talk is based on our works of refs. 1 and 2 collaborated with Y.B. Dai, C.S. Huang, M.Q. Huang and H.Y. Jin.

The study of the excited heavy mesons are interesting for the following reasons[3]. First, it may help us understand QCD more deeply in the nonperturbative aspect. Second, it is an alternative application of the heavy quark effective theory (HQET), in addition to that of ground state heavy mesons. Finally, it is particularly useful for tagging in CP experiments [4].

In this talk, we focus on the $L = 1$ excited charm mesons. It is heuristic to illustrate them in the following way, although it is not accurate (L is not a conserved quantity). For the ground state ($L = 0$) mesons which we are familiar with, there are two states with the J^P quantum numbers: $D(0^-)$ and $D^*(1^-)$. Their difference only lies in the heavy quark spin direction. For the $L = 1$ case, there are four states: $D_0^*(0^+)$, $D_1'(1^+)$, $D_1(1^+)$ and $D_2^*(2^+)$. In the constituent picture, D_0^* contains one heavy quark and one light quark with same spin direction which is opposite to that of the orbital angular momentum. D_1' is the same as D_0^* up to a heavy quark spin flip. D_1 is different from D_0^* by the light quark spin flip. And D_2^* is the same as D_1 except for the heavy quark spin direction.

These kinds of heavy mesons can be systematically studied by HQET[5]. In the heavy quark limit, the heavy quark SU(2) spin symmetry implies that the four excited heavy mesons can be grouped into two doublets (D_0^*, D_1') and (D_1, D_2^*)[6]. It is interesting to note that although D_1' and D_1 have same quantum numbers, they are clearly separated in the heavy quark limit. Their mixing is at the order of $1/m_c$, which can be studied in HQET. In the following, we present the HQET sum rule calculations for masses and strong decays of the excited charm mesons in the leading order.

The QCD sum rule[7] is a nonperturbative method rooted in QCD itself. The relevant interpolating current should be fixed. For the excited heavy

mesons, we wrote the currents as follows [1],

$$J = \frac{1}{\sqrt{2}} \bar{h}_v \Gamma D_\rho q \; , \tag{1}$$

with Γ denoting some γ matrices. h_v is the heavy quark field with velocity v in HQET. We find that

$$
\begin{aligned}
\Gamma &= -\gamma_t^\rho & \text{for} \quad D_0^* \; , \\
\Gamma &= \gamma_5 \gamma_t^\mu \gamma_t^\rho & \text{for} \quad D_1' \; , \\
\Gamma &= -\sqrt{\frac{3}{2}} \gamma_5 (g_t^{\mu\rho} - \frac{1}{3} \gamma_t^\mu \gamma_t^\rho) & \text{for} \quad D_1 \; , \\
\Gamma &= \frac{1}{2} (\gamma_t^\mu g_t^{\nu\rho} + \gamma_t^\nu g_t^{\mu\rho}) - \frac{1}{3} g_t^{\mu\nu} \gamma_t^\rho & \text{for} \quad D_2^* \; ,
\end{aligned}
\tag{2}
$$

where $\gamma_t^\mu \equiv \gamma^\mu - v^\mu \not{v}$ (In the rest frame of the meson, $\gamma_t^\mu = (0, \vec{\gamma})$) and $g_t^{\mu\nu} \equiv g^{\mu\nu} - v^\mu v^\nu$. We have shown that the current for one of the two 1^+ state does not couple to the other in the limit $m_Q \to \infty$. The related decay constant f is defined as

$$\langle 0|J^\dagger|D^{**}(v,\eta)\rangle \equiv f\eta \; , \tag{3}$$

where D^{**} stands for any of the four excited mesons with polarization vector η, which is the ground state of J (For more details, see ref. 1).

1 Masses

In HQET, the meson masses are expanded as

$$M = m_Q + \bar{\Lambda} + O(\frac{1}{m_Q}) \; , \tag{4}$$

where $\bar{\Lambda}$ denotes the meson masses defined in HQET. They are independent of heavy quark flavor. For the sake of calculating them, the Green's function is written as

$$\Gamma(\omega) = i \int d^4 x e^{ik \cdot x} \langle 0|\mathrm{T} J^\dagger(x) J(0)|0\rangle \; , \quad \omega = 2k \cdot v \; . \tag{5}$$

It can be expressed in the hadronic language,

$$\Gamma(\omega) = \frac{2f^2}{2\bar{\Lambda} - \omega} + \text{resonance} \; . \tag{6}$$

2

The Feynman diagrams can be found in ref. 1. The condensates are included up to dimension five. Here the usual duality hypothesis is used for the resonance contribution. After Borel transformation, the sum rules for $\bar{\Lambda}$'s are obtained then

$$
\begin{aligned}
\bar{\Lambda}\left(\begin{array}{c} 0^+ \\ 1^+ \end{array}\right) &= \frac{\frac{3}{8\pi^2}\int_0^{\omega_c} d\nu\, \nu^5 e^{-\nu/T}}{\frac{3}{4\pi^2}\int_0^{\omega_c} d\nu\, \nu^4 e^{-\nu/T} - m_0^2\langle\bar{q}q\rangle} \ , \\
\bar{\Lambda}\left(\begin{array}{c} 1^+ \\ 2^+ \end{array}\right) &= \frac{\frac{1}{32\pi^2}\int_0^{\omega_c} d\nu\, \nu^5 e^{-\nu/T} - \frac{1}{16\pi^2}\langle\alpha_s GG\rangle T^2}{\frac{1}{16\pi^2}\int_0^{\omega_c} d\nu\, \nu^4 e^{-\nu/T} - \frac{m_0^2}{3}\langle\bar{q}q\rangle - \frac{T}{8\pi}\langle\alpha_s GG\rangle} \ .
\end{aligned}
\tag{7}
$$

Imposing usual criterium for the upper and lower bounds of the Borel parameter T, we obtain the numerical results,

$$
\bar{\Lambda}\left(\begin{array}{c} 0^+ \\ 1^+ \end{array}\right) = 0.90 \pm 0.10 \text{ GeV} \ , \quad \bar{\Lambda}\left(\begin{array}{c} 1^+ \\ 2^+ \end{array}\right) = 0.95 \pm 0.10 \text{ GeV} \ , \tag{8}
$$

where the error comes from the uncertainty of $\omega_c \sim 0.5$ GeV.

2   Strong Decays

The pionic decays of heavy hadrons can be studied by combing heavy quark symmetry and chiral symmetry [8]. The excited heavy meson decays have been studied in this framework [9, 10]. We will not use soft pion approximation however, because the pion energy is about 500 MeV in this case. The decay widths are given by, *e.g.*,

$$
\begin{aligned}
\Gamma(D_0^* \to D\pi) &= \frac{3}{8\pi} g'^2 |\vec{p}_\pi| \ , \\
\Gamma(D_2^* \to D\pi) &= \frac{1}{20\pi} g^2 |\vec{p}_\pi|^5 \ ,
\end{aligned}
\tag{9}
$$

where sum over charged and neutral pion final states has been implied, and g' and g are universal quantities describing the decay amplitudes which we are going to calculate by QCD sum rules.

For this purpose, the Green's function is constructed as

$$
\tilde{\Gamma} = i \int d^4x\, e^{-ik\cdot x} \langle\pi(q)|\mathrm{T} J_{D^{(*)}}(x) J^\dagger(0)|0\rangle \ , \tag{10}
$$

3

with $J_{D^{(*)}}$ being the currents for $D^{(*)}$ mesons. The hadronic expression is

$$\tilde{\Gamma} = \frac{g^{(')} f_{D^{(*)}} f_{D^{**}}}{(2\bar{\Lambda}_{D^{(*)}} - 2v \cdot k)(2\bar{\Lambda}_{D^{**}} - 2v \cdot k')} + \frac{c}{2\bar{\Lambda}_{D^{(*)}} - 2v \cdot k} + \frac{c'}{2\bar{\Lambda}_{D^{**}} - 2v \cdot k'} + \text{res.} \ , \tag{11}$$

where $k' = k - q$. Instead of taking soft pion limit, we put $2v \cdot (k - k') = 2(\bar{\Lambda}_{D^{**}} - \bar{\Lambda}_{D^{(*)}})$. In the operator product expansion of HQET,

$$\tilde{\Gamma} = i \int_0^\infty dt e^{\frac{\omega t}{2}} \frac{1}{\sqrt{2}} \langle \pi(q) | T \bar{q}(vt) \Gamma_{D^{(*)}} \frac{1 + \not{v}}{2} \Gamma D_\rho q(0) | 0 \rangle \ . \tag{12}$$

What we need is $\langle \pi | \bar{q}_\alpha^a(x) D_\rho q_\beta^b | 0 \rangle$ which can be found in ref. 2. With

$$g_2 = \frac{f_\pi}{4} \ , h_1 = 0 \ , h_2 = -\frac{1}{12 f_\pi} \langle \bar{q}q \rangle \ , a_1 = -\frac{m_0^2}{8 f_\pi} \langle \bar{q}q \rangle \ , c_1 = \frac{1}{36} f_\pi m_1^2 \ ,$$

$$c_2 = -\frac{1}{24 f_\pi} \langle \bar{q}q \rangle \ , b_1 = \frac{a_1}{3} \ , d_1 = 0 \ , d_2 = -\frac{e_2}{3} + \frac{1}{12 f_\pi} \langle \bar{q}q \rangle \ , m_1^2 \simeq 0.2 \text{GeV}^2 \ , \tag{13}$$

and e_2 which is defined as

$$e_2 (n \cdot q)^3 = \frac{i}{2} \langle \pi^i(q) | \bar{q} \frac{\tau_i}{2} \gamma_5 \not{n} (n \cdot D)^2 q | 0 \rangle \ , \quad n^2 = 0 \ , \tag{14}$$

and is fixed from QCD sum rule [2]: $e_2 \simeq -0.015 \pm 0.002$ GeV, the final sum rules for g' and g are

$$g' f_{D_0^*} f_D = 2\{\bar{\Lambda}_{D_0^*}(3h_1 + 3h_2\Delta - g_2\Delta^2) - [-3b_1 + 3\Delta(c_1 + d_1)$$

$$+ \Delta^2(4c_2 + 2d_2 - e_2)](1 + \frac{2\bar{\Lambda}_{D_0^*}}{T})\} e^{\frac{2\bar{\Lambda}_{D_0^*}}{T}} \ , \tag{15}$$

$$g f_{D_2} f_D = 2\left[\bar{\Lambda}_{D_2} g_2 + (c_2 - e_2\Delta - d_2)(1 + \frac{2\bar{\Lambda}_{D_2}}{T})\right] e^{\frac{2\bar{\Lambda}_{D_2}}{T}} \ ,$$

where $\Delta = \bar{\Lambda}_{D^{**}} - \bar{\Lambda}_D$. The range of the Borel parameter is fixed by requiring higher $\frac{1}{T}$ term to be small in OPE for original sum rules, and T being not too large than $2(\bar{\Lambda}^1 - \bar{\Lambda})$ where $\bar{\Lambda}^1$ is the mass of first radial excitation in HQET. We obtain the numerical results

$$\begin{aligned} g' &\simeq 1.7 \pm 0.5 \ , \\ g &\simeq 4.6 \pm 1.0 \quad \text{GeV}^{-2} \ . \end{aligned} \tag{16}$$

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For discussion, our methods improve the calculations by sum rules in full QCD[11] which have problems of including contamination of the other 1^+ state, and using soft pion approximation. The $1/m$ corrections[12] and radiative corrections can be included systematically. The experiment data on D_2^* width[13] can be explained by assuming the pionic decay mode dominant. However, that of D_1 cannot be understood in this way. It maybe due to the mixing of two 1^+ states at the order of $1/m$ as well as that $D_1 \to D^{(*)}\rho$ have relatively large branching ratios.

Acknowledgments

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